Denoising images with Poisson noise statistics

Triet Le, tle@math.ucla.edu
Rick Chartrand, rickc@lanl.gov
Thomas J. Asaki, asaki@lanl.gov

An important task in mathematical image processing is image denoising. Many image denoising algorithms assume that the noise is normally distributed and additive. Many images, however, contain noise that satisfies a Poisson distribution. The magnitude of Poisson noise varies across the image, as it depends on the image intensity. This makes removing such noise very difficult. We use Bayes's Law to develop a new denoising algorithm, which removes Poisson noise while preserving image features that other methods remove.

The general idea behind most denoising methods is to regard a noisy image f as being obtained by corrupting a noiseless image u. The desired image u is then a solution of the corresponding inverse problem: which u could f be obtained from by corruption? Since there is generally more than one solution, most denoising procedures employ some sort of regularization. A very successful algorithm is that of Rudin, Osher, and Fatemi (ROF; [1]), which uses total-variation regularization. The ROF model regards u as the solution to a variational problem, to minimize the functional

$$F(u) := \int_\Omega |\nabla u| + \frac{\lambda}{2}\int_\Omega |u-f|^2, \qquad (1)$$

where Ω is the image domain and λ is a parameter to be chosen. The first term is a regularization term, the second a data-fidelity term. Minimizing $F(u)$ has the effect of diminishing variation in u, while keeping u close to the data f. The size of the parameter λ determines the relative importance of the two terms.

Like many denoising models, the ROF model is most appropriate for signal independent, additive, Gaussian-distributed noise. Many important data, however, contain noise that is signal dependent and Poisson distributed. A familiar example is that of radiography. The signal in a radiograph is determined by photon counting statistics, which are Poisson distributed. Removing noise of this type is a more difficult problem. Existing approaches proceed only under restrictive assumptions.

We use Bayes's Law to derive a model much like ROF, but with a data-fidelity term that is customized for Poisson noise. The result is to seek a minimizer of

$$E(u) := \int_\Omega |\nabla u| + \beta \int_\Omega (u - f\log u),$$

where β is a constant that determines the relative effect of the two terms.

We find the minimizer by gradient descent. That is, we start from some initial choice of u, then move in the opposite direction of the derivative of E. We thus replace u with $u - \alpha E'(u)$ for some constant α, and then repeat the process. At each step, the value of $E(u)$ will decrease. The process is continued until the minimum value is reached.

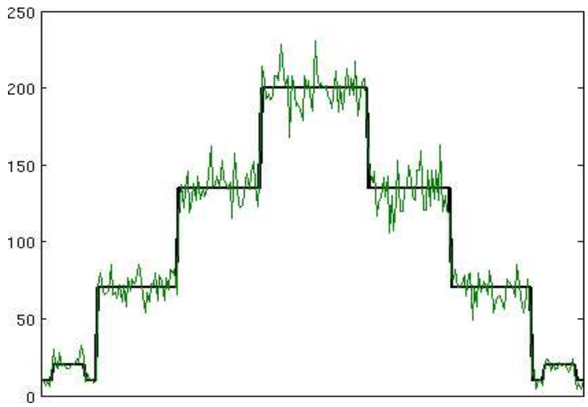
The derivative of the functional E is

$$E'(u) = -\operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right) + \frac{\beta}{u}(u-f).$$

Compare this with the derivative of the ROF functional (1):

$$F'(u) = -\operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right) + \lambda(u-f).$$

Notice that the two derivatives are similar, with λ replaced by the varying β/u, which depends on the reconstructed image u. This local variation of the regularization parameter is better suited for Poisson noise, because the expected noise increases with image intensity. Decreasing the value of the regularization parameter increases the denoising effect of the regularization term in the functional. This will happen where the values of u are larger, which is precisely where the noise level is larger. We thus have a model that is similar to ROF but with a self-adjusting parameter.



An example for comparison of our model with the ROF model. A cross-section of an image (black line) with Poisson noise added (green line). The greater the image intensity, the greater the local noise level.

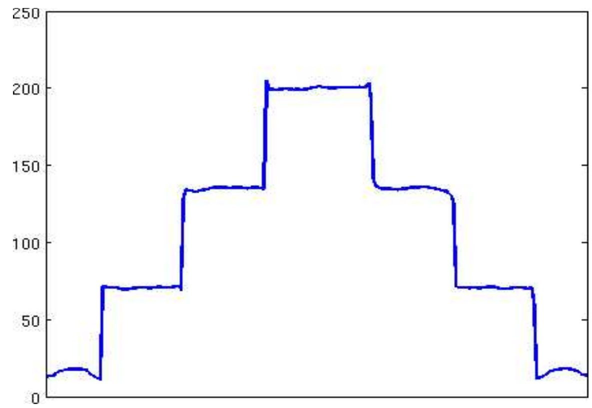
The figures show the results of applying ROF and our proposed model to a test image with Poisson noise. Cross-sections are shown for greater clarity. When ROF is used with the parameter λ as appropriate for the overall measured noise level, small-scale features are removed along with the noise. If the regularization strength is decreased (by increasing λ) to a level appropriate for the smaller-scale noise present in regions of lower image intensity, then the larger-scale noise is not removed. Our model removes noise at all scales, while preserving small-scale features in regions of small-scale noise.

Acknowledgements

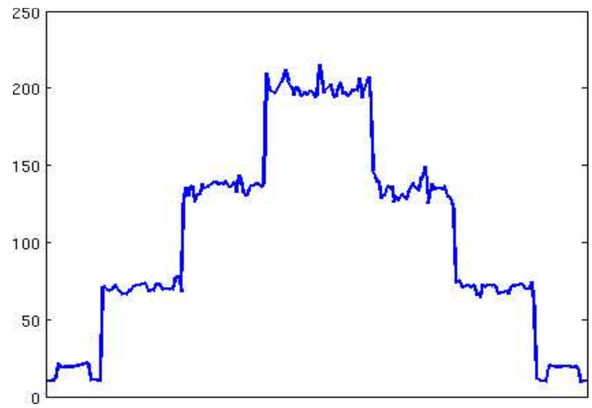
Los Alamos Report LA-UR-05-6185. Funded by the Department of Energy under contracts W-7405-ENG-36 and the DOE Office of Science's Advanced Scientific Computing Research (ASCR) Applied Mathematical Research program.

References

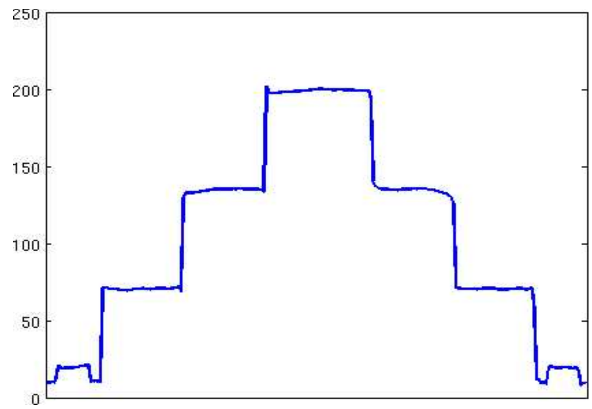
[1] L. RUDIN, S. OSHER, AND E. FATEMI. Nonlinear total variation based noise removal algorithms. *Physica D*, 60:259–268, 1992.



The ROF model removes noise, but washes out the bump feature on the sides.



Decreasing the regularization strength of the ROF model preserves the feature, but doesn't remove larger-scale noise.



Our model removes noise and preserves small-scale features.